Weather-driven model indicative of spatiotemporal power laws

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(Received 15 September 2006; published 26 January 2007)

In the traditional Drossel-Schwabl forest fire model (DS model), the frequency distributions of fire size and fire interval follow a power law and an exponential law, respectively. However, it is found that the frequency-interval distribution of actual forest fires is not exponential, but a power law with periodical fluctuations which may be caused by the daily cycle of weather parameters. Therefore, a weather driven forest fire model (WD model) is built considering actual hourly weather records, with which the fire igniting probability is calculated. The simulation results indicate that the frequency-interval distribution of the WD model agrees with that of actual forest fire data and, at the same time, the frequency-size distributions of the WD and the DS models are in accordance with each other. In the further analysis of the temporal property of weather data, it is found that the change of weather data also exhibits a power-law relation with periodic fluctuations, implying that the external driving from weather parameters is the essential reason for the power-law distribution of fire intervals. The results suggest that natural systems may be coupled with each other and that the decoupling of systems is important to identifying system characteristics.

DOI: 10.1103/PhysRevE.75.016109 PACS number(s): 05.65.+b, 89.75.Da, 87.18.Hf

I. INTRODUCTION

During the past several decades, self-organized criticality (SOC) introduced by Bak *et al.* [1,2] has been one of the research focuses. The SOC system has continuous phase transition whose dynamics turn the critical point into an attractor [1–4]. It evolves into a steady critical state irrespective of initial conditions and without fine-tuning of parameters. In the steady state, the frequency-size or frequency-intensity distribution of the dissipation events satisfies the power-law relation. Power-law distributions have been found in many actual systems and phenomena, including ecosystem [5], earthquake [6], World Wide Web [7–11], family names [12], rainfall [13–15], citation of papers [16], economic activities [17,18], population [19], biological species or patches [20,21], forest fires [22–34], etc.

Forest fires have attracted attention from different fields [22–34]. The forest fire model introduced by Drossel and Schwabl [22], i.e., the DS model, has been claimed to be self-organized critical and been used to explain actual forest fire distributions. It is found that forest fires in the USA and Australia [23–26], Italy [27], and China [28] exhibit good power-law distributions over many orders of magnitude, consistent with the model data. Considering the complexities of the initiation and propagation of forest fires, it is remarkable that the frequency-area statistics are very similar under a wide variety of environments [27].

However, in the traditional DS model, the time scales are essentially infinitely separated and the interoccurrence times are not quantified. At each sparking action in the DS model, the forest density is approximately a stable value, therefore the distribution of fire intervals exhibits exponential. From the phase-transition perspective, the critical slowing down, the divergence of relaxation times, and the disappearance of characteristic time scales are expected to be observed. Both solar flares, earthquakes, and forest fires have been found exhibiting power-law first-return-time or quiescent-time distributions [35–38]. And other studies [39,40] argue that temporal power law behaviors are based on the statistics of large events greater than some prescribed threshold, but dynamical temporal correlations among events of a sufficiently large size are inherent in SOC.

In order to explore the reason for the temporal power-law distribution of forest fires, in this paper, we study a weather driven model (WD model) built based on the DS model, and analyze the distribution characteristics of weather parameters. The advantages of forest fire study are the relative simple dynamics and detailed records of both forest fires and their impact factors. It is convenient to analyze and verify the spatial and temporal variables.

II. DATA, MODEL, AND DISCUSSION

We analyzed five years' (1996–2000) countrywide fire records in Japan, in which all fires with a burnt area greater than 100 m^2 are recorded. Our main focus is on the system-wide distribution of fire size and fire interval, i.e., duration between the triggering times of two successive fires. First, the frequency-interval distributions of the DS model and actual data are compared. It is found that the frequency-interval distribution of actual forest fires does not follow exponential relation as the DS model predicts, but behaves as a power law spanning for 2–3 decades with periodic fluctuations, as shown in Fig. 1.

The differences between the DS model and actual fire data may be due to two reasons. The first one is the threshold effect [39,40]. In the fire records, only fires greater than

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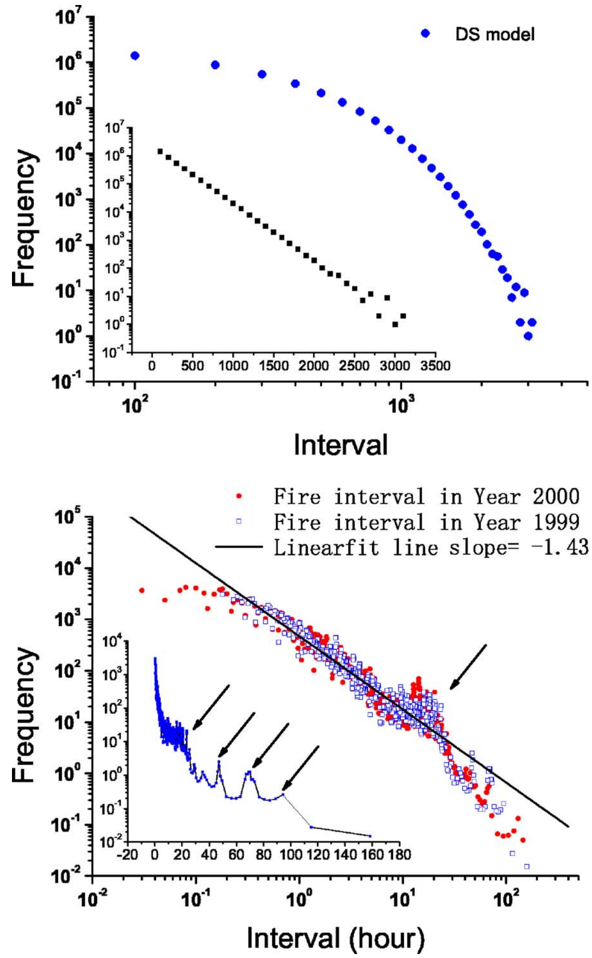

FIG. 1. (Color online) Comparison between the DS model and actual forest fires. It is shown that the frequency-interval distributions of the DS model and actual data differ qualitatively from each other, i.e., the modeled one satisfies the exponential law and the actual one satisfies the power law with periodic fluctuations. The insets of the figures are log-linear plots of the same data.

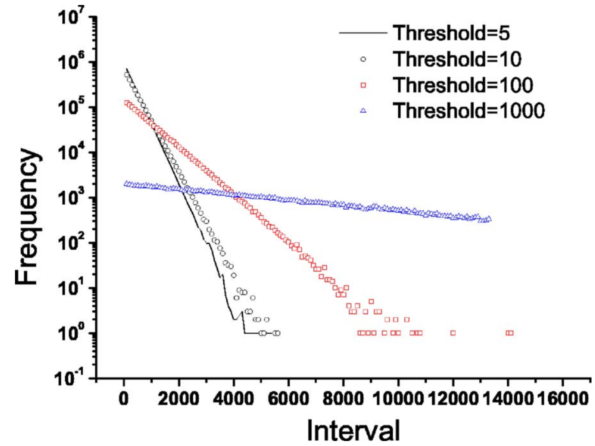

FIG. 2. (Color online) Threshold effects in the DS model. The frequency-interval distributions obey the exponential law, not the power law, indicating that the threshold effect is not the reason for the difference between the DS model and the actual data.

100 m^2 are recorded, those smaller than this threshold are difficult to detect and record because of the complex conditions in the forest and the large scale of the system. Above this threshold, forest fires might have both spatial and temporal correlations with each other. The second one is the external driving. As shown in Fig. 1, the fire interval distribution curve has periodic fluctuations deviating from the power law. The fluctuations have a time interval of about one day, reflecting the daily cycle of influence factors such as weather parameters, which may impact the temporal distribution of forest fires.

To confirm the first possible reason, i.e., threshold effect, we analyze the DS model [22]. The model is a cellular automata model. The forest is represented with a $L \times L$ lattice, in which trees grow with a probability p and fire occurs with a lesser probability $f \ll p$. A burning tree will ignite all its neighboring trees (except those already burnt) so that a forest cluster will burn down in a single event.

We set different thresholds for the burnt area. Fires with burnt area smaller than the threshold are ignored in the fire distribution analysis. It is shown in Fig. 2 that the fire interval distribution is exponential whether a threshold is applied or not, which implies that the threshold effects or the precision of statistics is not the reason for temporal power law distribution of forest fires.

There is a variety of weather parameters such as humidity, temperature, wind speed, sunshine time, and so on. Although these parameters influence forest fires synthetically, we consider them one by one in order to explore the basic relation between igniting probability and weather. Thus a weather driven forest fire model (WD model) is introduced based on the DS model. The rules of the new model are as follows.

(1) A burning tree becomes an empty site.
(2) A tree becomes a burning tree if at least one of its nearest neighbors is burning.
(3) At an empty site a tree grows with probability p.
(4) A tree without a burning nearest neighbor becomes a burning tree with probability $f = f(h)$; here h is the weather parameter.

The only difference between the two models is the igniting rule or rule (4). In the WD model, fire is not ignited randomly, but with a probability related to the weather parameter. How to get the weather parameter and its relation to the igniting probability is a core work in the model simulation and analysis.

One of the most important weather parameters that influence forest fire is relative humidity. The dependence of the igniting probability on relative humidity has been studied by statistics [41], as shown in Fig. 3. The relation is a monotonic decreasing curve and, as an estimation, can be fitted as a polynomial, as shown in formula (1)

$$f(h) = 3.04 \times 10^{-4} - 6.71 \times 10^{-6} h + 3.668 \times 10^{-8} h^2. \tag{1}$$

The igniting probability can be calculated with this formula on the condition that the relative humidity value is known. Figure 4 illustrates the hourly relative humidity records in the Chiba weather station of Japan in 1999. The

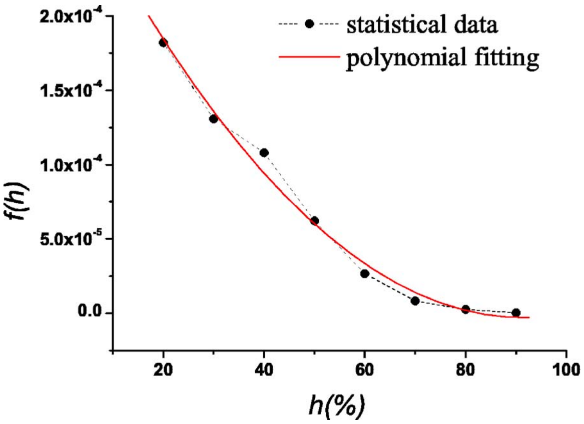

FIG. 3. (Color online) Relation between relative humidity and igniting probability. The data is obtained through analysis of forest fire records in countrywide Japan in 1999. The polynomial fitting is $f(h)=3.04\times10^{-4}-6.71\times10^{-6}h+3.668\times10^{-8}h^2$, with r square (COD)=0.99051, SD=7.86184×10^{-6}, and p value<0.0001.

average value of relative humidity undergoes monthly and seasonal changes that may affect the igniting probability of forest fires and thus affect the distribution of fire intervals. The humidity data are input into the WD model and used to calculate the igniting probability by formula (1).

Simulation results of the WD model are shown in Figs. 5 and 6. It is demonstrated that the distribution of the fire area obtained with the WD model satisfies the power-law relation, which has the same characteristic as that of the DS model. However, the interval distribution of the WD model does not follow exponential law as the DS model, but a power law with periodic fluctuations, of which the period is as large as about 24 hours or one day, coinciding well with actual forest fire data (see Fig. 1). It seems clear that the driving of weather parameters is the basic reason for the power-law

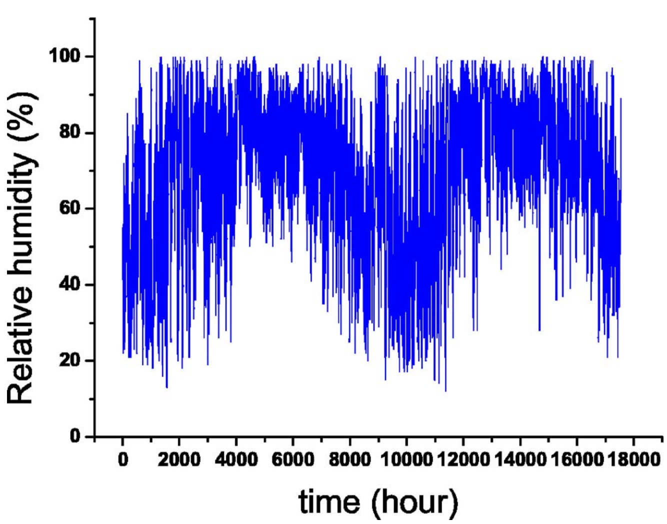

FIG. 4. (Color online) Relative humidity records in a Chiba weather station from 1999 to 2000. The changes of humidity with time behave themselves with complex shapes and annual cycle. The position of Chiba station is North latitude 35°36′, East longitude 140°6.5′. The humidity data used here is from http://www.data.kishou.go.jp/etrn/index.html

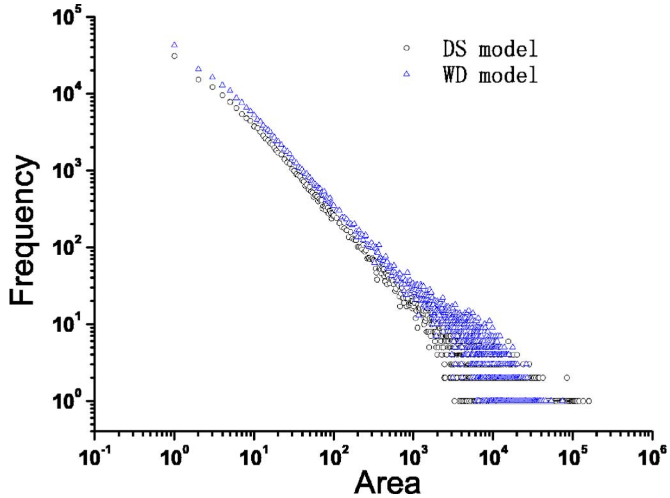

FIG. 5. (Color online) The frequency-area distributions of the DS model and the WD model. The two distributions all obey the power law and almost overlap with each other. The parameters used in the DS model are $f/p=1/1000$. Those of the WD model are $f/p=f(h)=3.04\times10^{-4}-6.71\times10^{-6}h+3.668\times10^{-8}h^2$, $n=50$ time steps per hour.

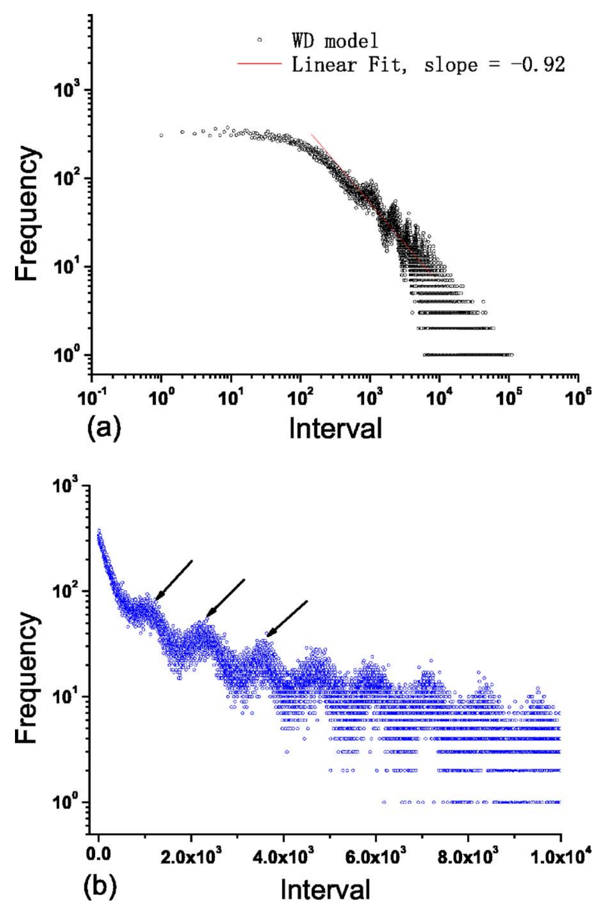

FIG. 6. (Color online) The frequency-interval distributions of the WD model. The relation satisfies the power law with periodic fluctuations. (a) Log-log plot; (b) log-linear plot.

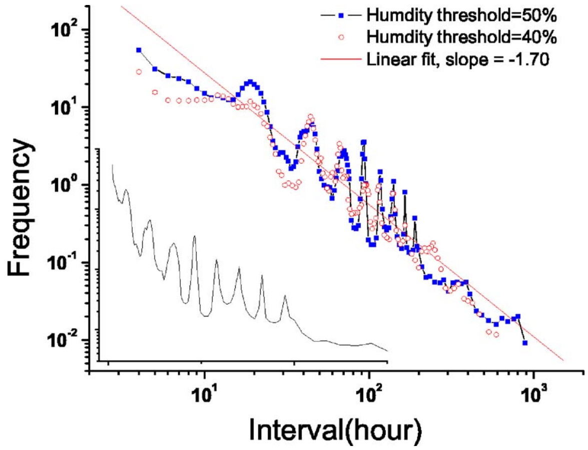

FIG. 7. (Color online) Power-law frequency-interval distribution of actual humidity data. The thresholds are 50% and 40%. It is shown that the distributions corresponding to different threshold values exhibit similar characteristics, i.e., power law with periodic fluctuations. The inset is the log-linear plot of the same data (threshold=50%).

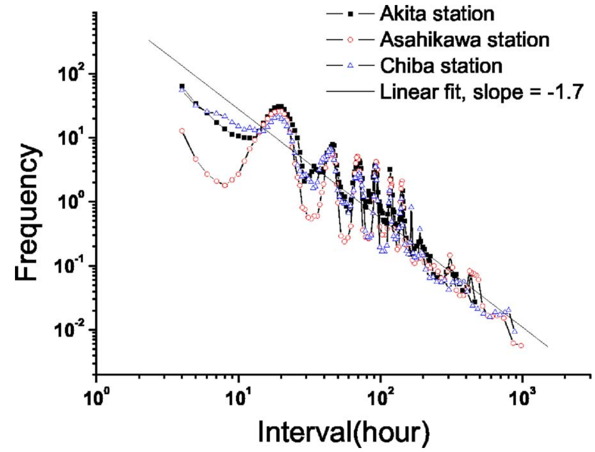

FIG. 8. (Color online) Steadiness of power-law distribution of humidity data. Humidity records from different weather stations exhibit similar characteristics, i.e., power-law frequency-interval distribution with similar exponential values. The threshold used is 50%.

distribution and the periodic fluctuation of the fire interval. To find out the underlying mechanism of weather driving, we analyze the temporal change of humidity records.

If the weather data are converted into binary data, i.e., (0, 1) sequence, with an arbitrary given threshold, we can analyze their temporal change in a simple way. If the humidity value is greater than the threshold, it is converted to "0," and otherwise, "1." The value 0 represents the passive state during which fire cannot occur. The value 1 means the active state during which fire occurs easily. The length of each active period will be proportional to that of the fire interval. It is found that the size distribution of active lengths satisfies a good power law with periodic fluctuations, as shown in Fig. 7. The results of different threshold, i.e., 50% and 40%, show similar distribution behaviors. The similarity between active length distribution of humidity data and fire interval distribution confirms that the power-law distribution of fire intervals is due to the external driving of weather parameters. Although the exponential value of the two types of distribution is not equal, the qualitative characteristics are the same.

Moreover, it is observed in Fig. 8 that the humidity data measured in different weather stations have similar distribution properties of both quantitative and qualitative aspects. The three weather stations are located in different area of Japan, i.e., Akita at North latitude 39°43′ East longitude 140°5.9′, Asahikawa at North latitude 43°45.4′ East longitude 142°22.3′, and Chiba North latitude 35°36′, East longitude 140°6.5′. Despite these differences, the distribution characteristic is similar. It is confirmed again that the external driving of weather parameter is the key reason for the power law and periodic fluctuations of frequency-interval distribution of forest fires.

It is found that the fluctuations of humidity display long-range correlation, i.e., power-law behavior [42], indicating that the humidity has spatial-temporal power law distributions. At the same time, the rainfall also exhibits spatial-temporal power law distributions [14], and the exponent is similar to that of humidity. The humidity and rainfall might be coupled with each other, just like the coupling between humidity and forest fires. Such a kind of system coupling may be popular in natural systems, adding to the difficulty of identifying system characteristics.

III. CONCLUSIONS

In this paper, the temporal scaling of forest fires has been connected with internal dynamics and external driving. For the DS forest fire model, the fire interval distribution is exponential-like, whether or not considering threshold effects. After taking into account the influence of typical weather data, i.e., humidity, the results of the WD model agree with actual forest fire records that both of them exhibit power-law size and time interval distributions.

Through the study of forest fire models and actual fire data, it seems clear to us that the external driving of weather, instead of internal dynamics of forest fire, is the main reason for the power-law scaling of fire intervals. The periodic fluctuation is due to the daily weather cycle.

For a forest fire system, the driving comes from the weather system and the distribution of the weather parameter follows the power-law relation. Other complex systems, e.g., earthquake or sun flare, might also be influenced by external factors that generate temporal power-law distributions. This kind of system coupling might be common in nature and the behaviors of a system might be mixed with those of other systems. So how to identify system behaviors from those of others, or decoupling of system behaviors, may be an interesting research subject.

ACKNOWLEDGMENTS

We acknowledge the support from the National Natural Science Foundation in China (30400344) and Special Fund for Major National Basic Research Project (973) in China (2001CB409600).

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